

# FREQUENCY STEERABLE TRANSDUCERS WITH 360° DIRECTIVITY FOR GUIDED WAVES INSPECTIONS IN PLATE-LIKE STRUCTURES

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**Abstract.** Directional Transducers for Guided Waves generation and sensing are achieved by patterning the piezoelectric material lay-out and the electrodes. The peculiar electrodes' shape produces a spatial filtering effect which is frequency-dependent, so that a direct relationship can be established between the direction of propagation (wavenumber) and the spectral content of the transmitted/received signal. This kind of transducer has been named Frequency Steerable Acoustic Transducers (FSATs). FSAT dedicated signal processing algorithms enable localization of acoustic events in 2D spaces by processing two differential output of a single device, i.e. using only two acquisition channel thus allowing for a drastic hardware simplification and cost reduction of Guided wave based systems. In this work, a new generation of FSATs is proposed, which allows steering the wave direction of propagation in the whole angular range [0°; 360°] with improved directivity and sidelobes reduction.

## Introduction

The integration of SHM technologies based on Guided Waves (GW) inspections in real aircrafts is nowadays limited. In fact, these systems are usually based on bulky instrumentation and numerous connecting cables for data and power transfer between a dense network of transducers and the central data processing unit. As a result, such systems hardly meet aircraft design standards (non-structural weight increment and necessity of creating holes for the wiring). Drastic hardware simplification and cost reduction of GW-based systems can be achieved by using transducers that present inherent directional capabilities, exhibiting preferential radiation/sensing directions. In particular, directional GWs generation and sensing can be achieved by varying the load distribution through patterned piezoelectric material lay-outs [1][2]. Such devices, known as Frequency steerable acoustic transducers (FSATs), have a shape that produces a spatial filtering effect that is frequency-dependent, so that a direct relationship can be established between the direction of propagation (wavenumber) and the spectral content of the transmitted/received differential signals. For plate like structures, such relation can be established for a given guided mode starting from the knowledge of wavenumber-frequency dispersion curves. Arbitrary directional scanning within the [0°; 180°] angular range was demonstrated with FSATs in a recent work [3]. However, in the first practical realizations of FSATs, two main limitations appeared: i) just a relatively rude approximation of the desired directivity has been achieved, this means that strong radiations occur even in directions other than the



desired one; ii) waves are excited or sensed contemporarily in one direction and in the opposite direction (180° ambiguity).

In a recent paper, a novel strategy to improve the FSAT directivity was proposed [4], while in this work, a new generation of FSATs is proposed, which allows to overcome the above mentioned 180° ambiguity. Such limitation is eliminated by combining the information of 2 differential signals rather than one, as happened for the 1st generation FSATs. The result of this combination is a signal whose spectrum peaks at a frequency “f<sub>d</sub>”. The piezoelectric load distribution of the 2nd generation FSAT is designed so that the value of “f<sub>d</sub>” varies as a function of the wave direction of propagation in the whole angular range [0°; 360°].

The effectiveness of the novel transducer technology is shown through a numerical validation with application to defect monitoring in an Aluminum plate.

## 2. The Frequency Steering Transducer Design Process

Following the theory presented in [3], the voltage measured by a piezoelectric transducer and due to an incident wave mode  $m$  can be written as:

$$V_m(\omega, \theta) = jU_m(\omega, \theta)H(\theta)k_m(\omega, \theta)D(k_{1,m}(\omega, \theta), k_{2,m}(\omega, \theta)) \quad (1)$$

where  $\theta$  is the direction of arrival of the incident wave mode  $m$ ,  $U_m$  represents the amplitude of the wave mode at the considered angular frequency  $\omega$ ,  $H$  is a function of the piezo-structure system,  $k_m$  is the angular wavenumber for the considered wavemode (while  $k_{1,m}$  and  $k_{2,m}$  are its components along the orthogonal directions 1 and 2), and  $D$  is a function (namely, the *Directivity function*) of the wavenumber which can be computed as:

$$D(k_1, k_2) = \int_{\Omega} e^{j(k_1x_1+k_2x_2)} f(x_1, x_2) dx_1 dx_2 \quad (2)$$

i.e. the 2D-Fourier Transform of the weight function (or *load distribution function*)  $f(x_1, x_2)$ , which represents the effect of the piezoelectric patches shapes and polarizations.

Being  $D$  and  $f$  a Fourier pair, it has been shown in [3] that it is convenient to design the desired directivity function in  $D$  and then apply the Fourier transformation to obtain the piezo-transducer geometry which actually produces such directional behaviour.

More specifically, when  $D$  is constructed so that

$$D(k_1, k_2) = j \frac{1}{N} \sum_{n=1}^N [\text{sinc}(a|k - \gamma_n|) - \text{sinc}(a|k + \gamma_n|)] \quad (3)$$

where  $k = (k_1, k_2)$  and  $\gamma_n = (\gamma_{1,n}, \gamma_{2,n})$  are wavevectors defined such that:

$$\gamma_{1,n} = \left[ (k_{max} - k_{min}) \frac{\theta_n - \theta_{min}}{\theta_{max} - \theta_{min}} + k_{min} \right] \cos(\theta_n) \quad (4)$$

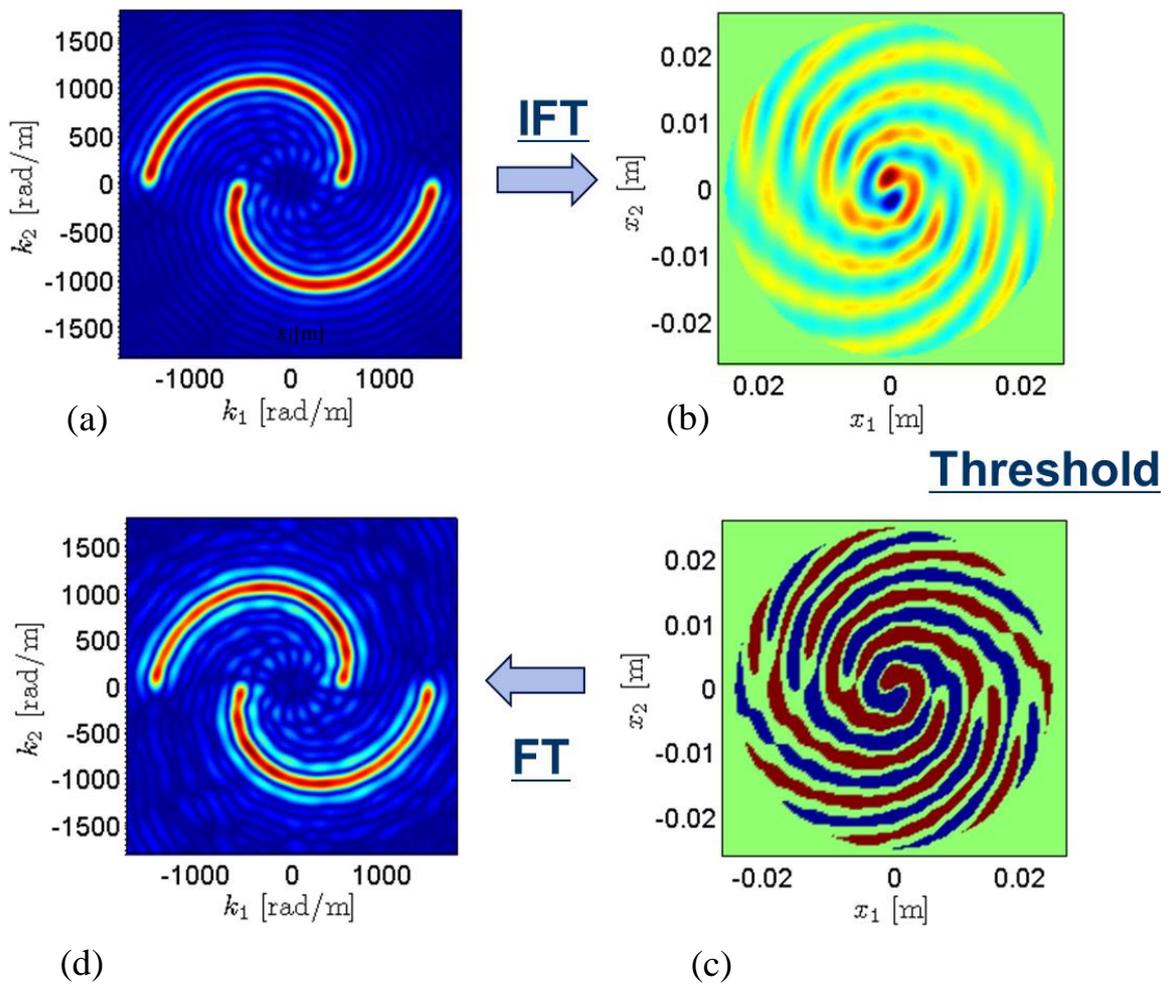
$$\gamma_{2,n} = \left[ (k_{max} - k_{min}) \frac{\theta_n - \theta_{min}}{\theta_{max} - \theta_{min}} + k_{min} \right] \sin(\theta_n) \quad (5)$$

It is thus possible to have different wavenumber maxima  $\gamma_n$  for each angle  $\theta_n$ . In its turn, each  $\gamma_n$  can be associated to an angular frequency on the basis of the dispersion relation  $k_m(\omega)$  which is specific to the considered waveguide. This means that the frequency response of a transducer characterized by the Directivity function described in Eq. (3) shall have a peak in correspondence to the angular frequency associated to the direction of propagation of the

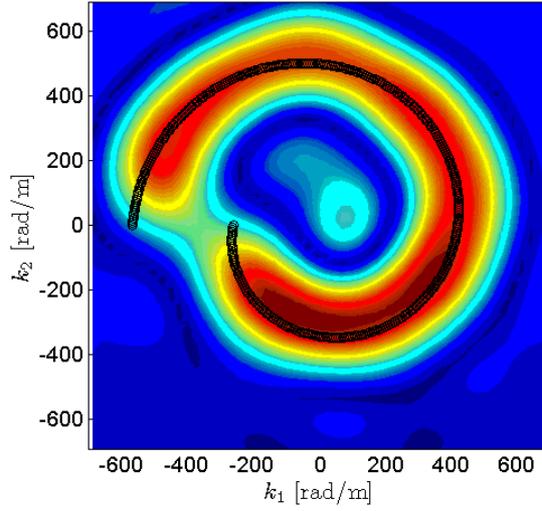
incident wave. Thanks to the reciprocity of the piezo sensing and actuation mechanism, it is also possible to generate a wave mode with a specific direction of propagation, by exciting the transducer with a waveform whose frequency content peaks at the associated angular frequency.

Unfortunately, the load distribution  $f$  which corresponds to the Fourier Transform of (3) cannot be realized in practice, as it would require continuously modulated amplitude. To tackle this problem, it is possible to quantize  $f$  so that to  $f$  is attributed the value  $+1$  when  $f > \varepsilon$  (being  $\varepsilon$  an arbitrary threshold),  $0$  for  $-\varepsilon < f < \varepsilon$ , and finally  $-1$  for  $f < -\varepsilon$ . Such approximation of the load distribution produces just a relatively rough approximation of the desired spiral-shaped directivity. An exemplification is reproduced in Pic. 1, where a sample FSAT radiation pattern is plotted. The side lobes in the wavenumber domain directly affect the radiation pattern of the transducer. It can be seen that strong radiations occur even in directions other than the desired one.

Another relevant limitation is due to the fact that the directivity function is symmetrical with respect to the origin, this means that a given waveform is excited both in a given direction and in its opposite one, the same  $180^\circ$ -ambiguity holds in sensing, to discriminate the direction of arrival of the incoming waves.



**Fig. 1.** FSAT transducer design: (a) Spiral-shaped load distribution in the wavenumber domain. (b) Associated spatial distribution. (c) and (d) effect of the quantization procedure on the directivity.



**Pic. 2.** Directivity function of the 2<sup>nd</sup> generation FSAT transducer.

### 3. A novel design strategy for FSATs

In this paper, a new design strategy for FSATs is proposed which is based on shaping the directivity function as a 360°-spiral, as schematically depicted in Pic. 2. More specifically, the Directivity function is given by the following formula:

$$D(k_1, k_2) = \frac{1}{N} \sum_{n=1}^N Ker(k_1 - \gamma_{1,n}, k_2 - \gamma_{2,n}) \quad (6)$$

where  $N$  is the number of kernel functions  $Ker(k_1 - \gamma_{1,n}, k_2 - \gamma_{2,n})$  replica used to synthesize the directivity function. In the proposed approach, Gaussian kernels are used:

$$Ker(a_1, a_2) = 2\pi \left(\frac{b}{2}\right)^2 \exp\left(-\left(\frac{b}{2}\right)^2 (a_1^2 + a_2^2)\right) \quad (7)$$

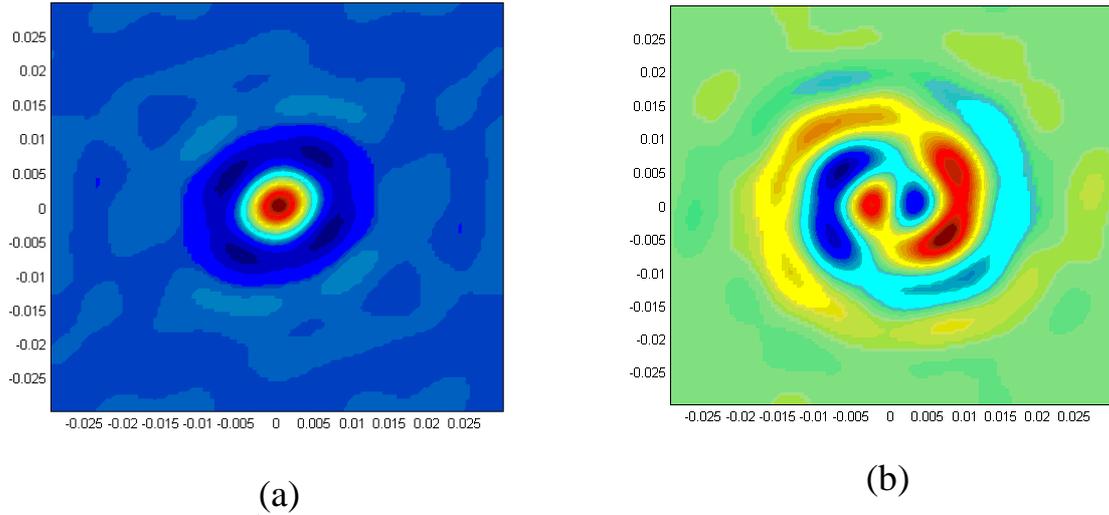
Where  $b$  is an arbitrary parameter, while  $\gamma_{1,n}$  and  $\gamma_{2,n}$  are given by:

$$\begin{cases} \gamma_{1,n} = k_{1,m}(\omega_n, \theta_n) \\ \gamma_{2,n} = k_{2,m}(\omega_n, \theta_n) \end{cases} \quad (8)$$

with  $\omega_n$  e  $\theta_n$  selected so that:

$$\begin{cases} \theta_n = \frac{2\pi}{N}n \\ \omega_n = \frac{\omega_{max} - \omega_{min}}{N}n + \omega_{min} \end{cases} \quad (9)$$

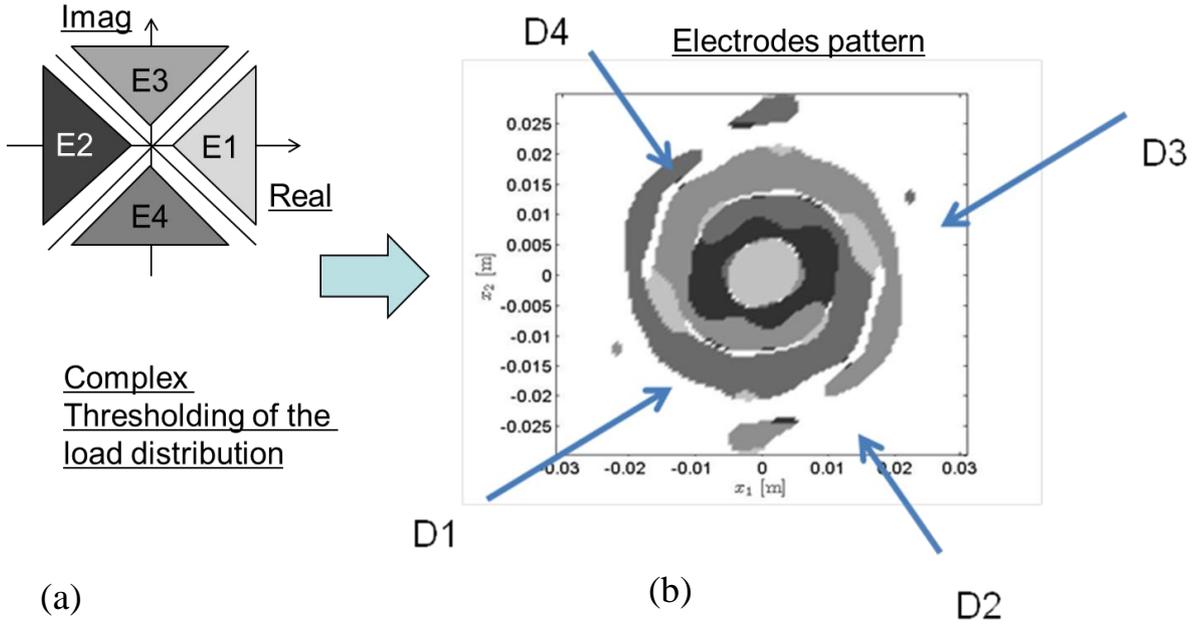
The load distribution  $f$  which corresponds to the Fourier Transform of (6) is a complex function. In particular, the imaginary and real load distribution corresponding to the directivity function of Pic 2 are depicted in Pic 3(a) and (b), respectively.



**Pic. 3.** Complex load distribution corresponding to the Directivity function depicted in Pic. 2: imaginary part (a); real part (b).

A complex thresholding function is used to determine the shapes of the piezoelectric transducer electrodes. Such procedure is schematically depicted in Pic. 4. Each point of the spatial domain of the load distribution is associated to 4 different electrodes E1-E4, more specifically:

- when  $Re(f(x_1, x_2)) > |Im(f(x_1, x_2))| + \delta$  the point is attributed to E1
  - when  $Re(f(x_1, x_2)) < -|Im(f(x_1, x_2))| - \delta$  the point is attributed to E2
  - when  $Im(f(x_1, x_2)) > |Re(f(x_1, x_2))| + \delta$  the point is attributed to E3
  - when  $Im(f(x_1, x_2)) < -|Re(f(x_1, x_2))| - \delta$  the point is attributed to E4
- where  $\delta$  is an arbitrary positive value.

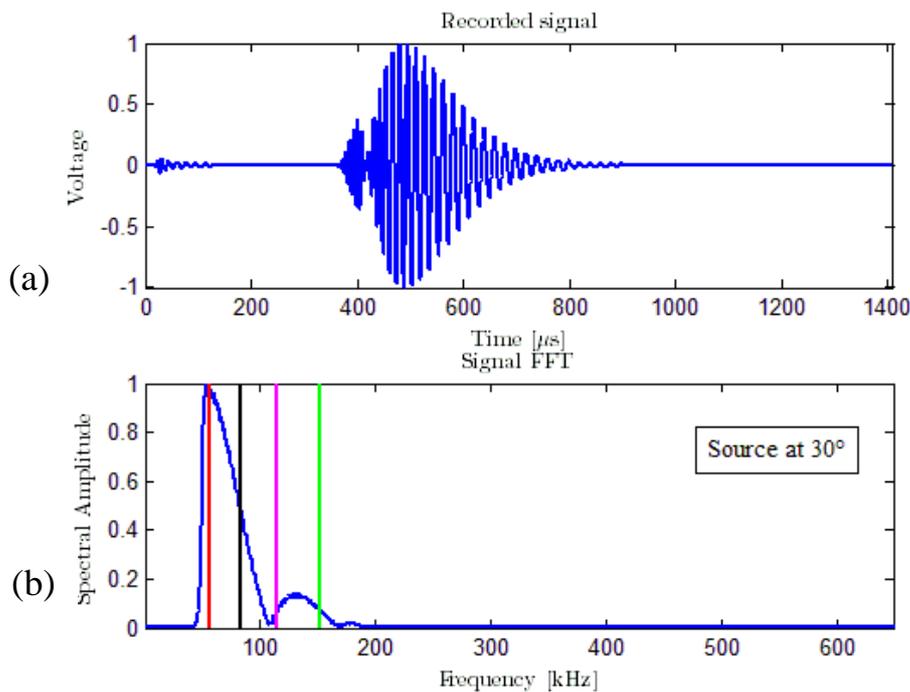


**Pic. 4.** Quantization strategy for the complex valued load distribution (a); 2<sup>nd</sup> generation FSAT electrodes shapes and directions of propagation of the incoming waves simulated in the result section (b).

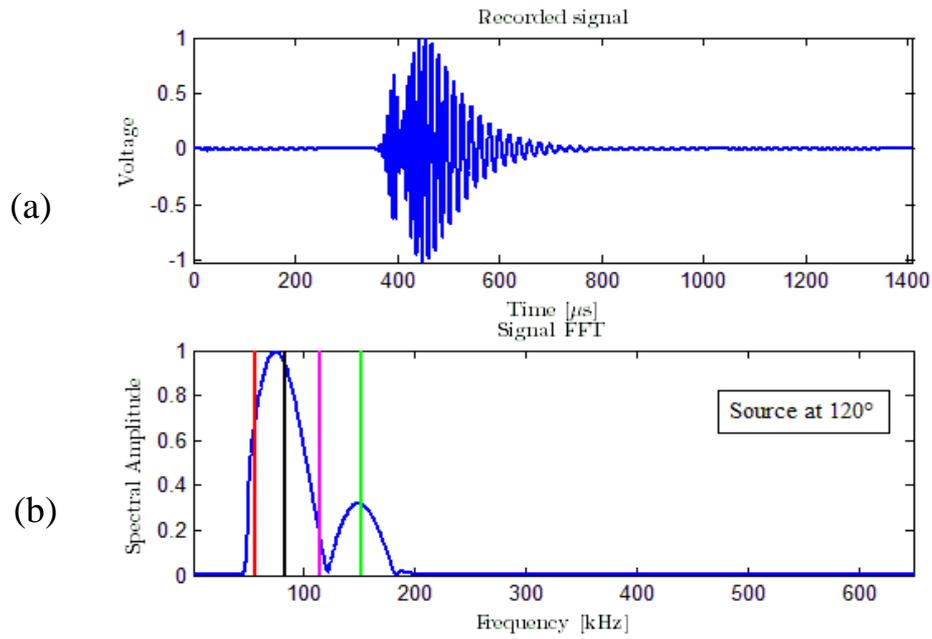
#### 4. Results

The FSAT design procedure described in the previous section was tested with numerical simulations. More specifically, the propagation of Lamb waves in an aluminium plate 3 mm thick was simulated. Four sample waveforms acquired by the FSAT transducer of Pic. 4 are depicted in Pic. 5-8. The acoustic waves are generated by 4 impulsive sources placed at 4 different angular positions ( $30^\circ$ ,  $120^\circ$ ,  $210^\circ$  and  $300^\circ$ , respectively) with respect to the transducer itself. It is worth noting how the frequency spectrum of the four signals is substantially different because it peaks in different positions, which are close to the nominal values (pointed by the vertical lines) imposed by the design procedures.

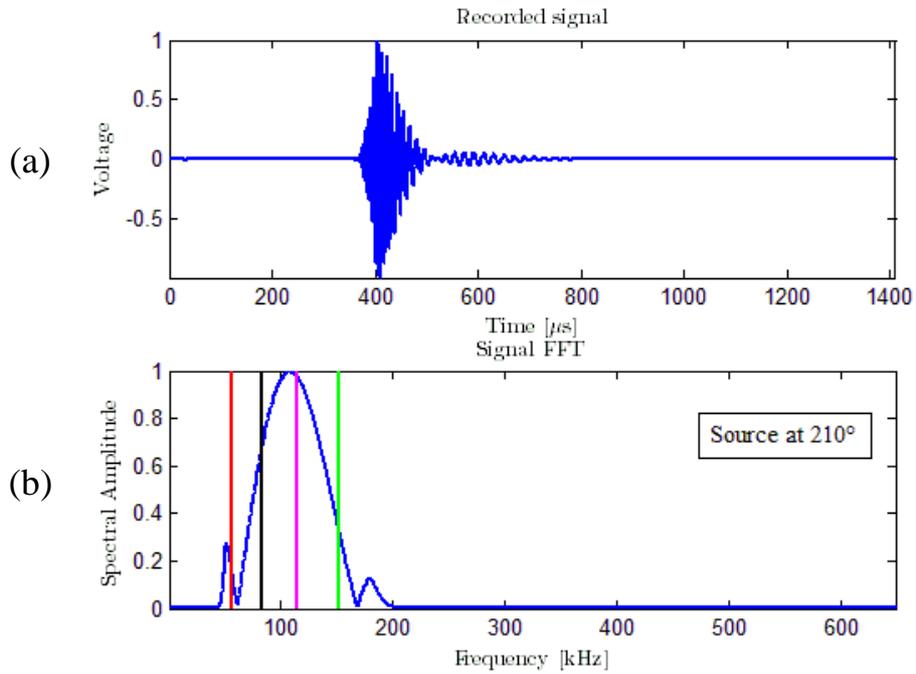
The simulated results show that undesired side lobes in the spiral shaped directivity function may appear. As shown in [4], such sidelobes can be reduced by applying more sophisticated thresholding procedures in order to enhance the angular resolution performance of FSAT-based inspection methods.



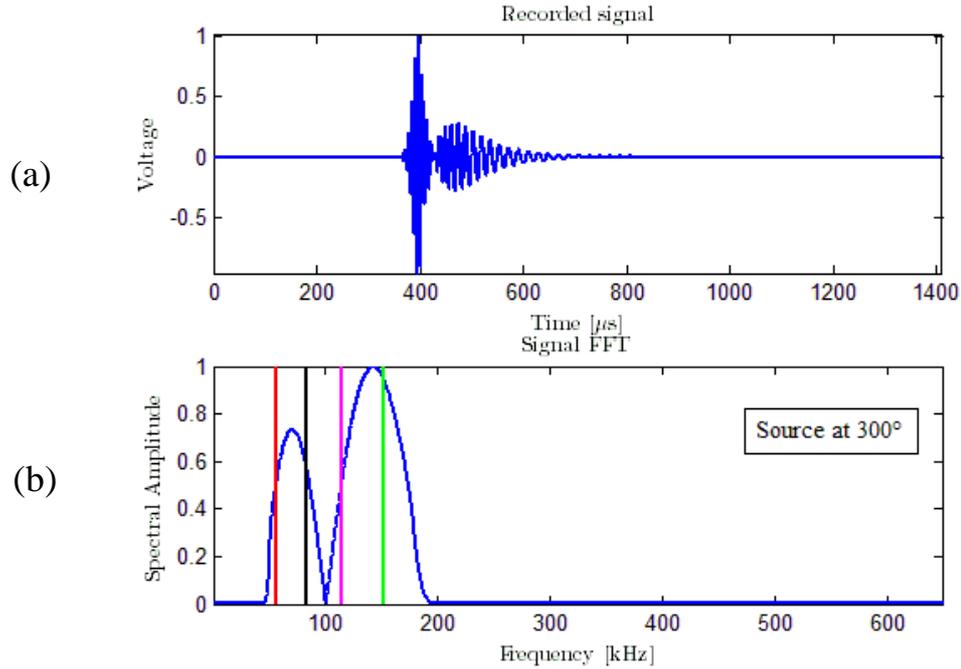
**Pic. 5.** Time history and frequency spectrum of the signal acquired by the FSAT transducer when excited by an impulsive source placed at  $30^\circ$



**Pic. 6.** Time history and frequency spectrum of the signal acquired by the FSAT transducer when excited by an impulsive source placed at 120°.



**Pic. 7.** Time history and frequency spectrum of the signal acquired by the FSAT transducer when excited by an impulsive source placed at 210°.



**Fig. 8.** Time history and frequency spectrum of the signal acquired by the FSAT transducer when excited by an impulsive source placed at  $300^\circ$ .

## 5. Conclusions

In this paper a novel strategy to realize Frequency Steerable Transducers was presented. Thanks to this strategy, a complex piezoelectric load distribution was designed, suitable to steer the acoustic beam over the whole ( $360^\circ$ ) angular range. Initial FSAT design attempts based on the local variation of piezoelectric material density have been made to increase the transducer directivity and reduce sidelobes. As further developments, we are planning to use dithering techniques to enhance the accuracy of the Directivity function approximation.

## References

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